The Effect of a Magnetic Field Dependent Viscosity on the Thermal Convection in a Ferromagnetic Fluid in a Porous Medium

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The effect of the magnetic field dependent (MFD) viscosity on the thermal convection in a ferromagnetic fluid in the presence of a uniform vertical magnetic field is considered for a fluid layer in a porous medium, heated from below. For a ferromagnetic fluid layer between two free boundaries an exact solution is obtained, using a linear stability analysis. For the case of stationary convection, the medium permeability has a destabilizing effect, whereas the MFD viscosity has a stabilizing effect. In the absence of MFD viscosity, the destabilizing effect of magnetization is depicted, but in its presence the magnetization may have a destabilizing or stabilizing effect. The critical wave number and critical magnetic thermal Rayleigh number for the onset of instability is determined numerically for sufficiently large values of the magnetic parameter M_1. Graphs are plotted to depict the stability characteristics. The principle of exchange of stabilities is valid for a ferromagnetic fluid heated from below and saturating a porous medium.

Key words: Ferromagnetic Fluid; Magnetic Field Dependent Viscosity; Thermal Convection; Porous Medium; Vertical Magnetic Field.

1. Introduction

Ferromagnetic fluids are obtained by suspending submicron sized particles of magnetite in a carrier such as kerosene, heptane or water. These fluids not found in nature, behave as a homogeneous medium and exhibit interesting phenomena. The method of forming ferrofluids was developped in the 1960s. For there wide ranges of application see [1 – 5].

An authoritative introduction to the research on magnetic liquids has been given by Rosensweig [6]. This monograph reviews several applications of heat transfer through ferrofluids. One such phenomenon is enhanced convective cooling having a temperature-dependent magnetic moment due to magnetization of the fluid. This magnetization depends on the magnetic field, temperature and density of the fluid. Any variation of these quantities can induce a change of the body force distribution in the fluid. This leads to convection in ferromagnetic fluids in the presence of magnetic field gradient. This mechanism is known as ferroconvection, which is similar to Bénard convection (Chandrasekhar [7]). The convective instability of ferromagnetic fluids heated from below in the presence of a uniform vertical magnetic field has been considered by Finlayson [8]. Thermoconvective stability of ferrofluids without considering buoyancy effects has been investigated by Lalas and Carmi [9], whereas Shliomis [10] analyzed the linearized relation for magnetized perturbed quantities at the limit of instability. The stability of a static ferrofluid under the action of an external pressure drop has been studied by Polevikov [11], whereas the thermal convection in a ferrofluid has been considered by Zebib [12]. The thermal convection in a layer of magnetic fluid confined in a two-dimensional cylindrical geometry has been studied by Lange [13]. A detailed account of magnetoviscous effects in ferrofluids has been given in a monograph by Odenbach [14].

There has been a lot of interest, in recent years, in the study of the breakdown of the stability of a fluid layer subjected to a vertical temperature gradient in a porous medium and the possibility of convective flow. The stability of flow of a fluid through a porous medium taking into account the Darcy resistance was considered by Lapwood [15] and Wooding [16]. A porous medium of very low permeability allows us to use the Darcy model (Walker and Homsy [17]). This is because for a medium of a very large stable particle suspension, the permeability tends to be small, justify-

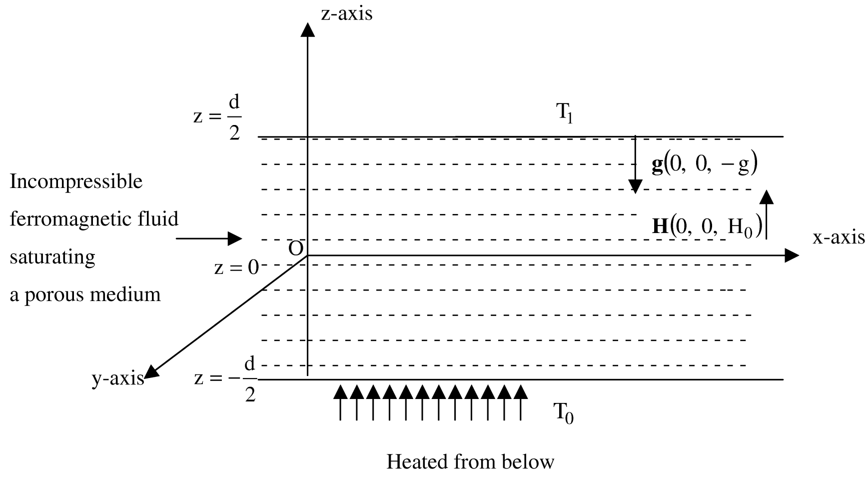

Fig. 1. Geometrical Configuration.

ing the use of the Darcy model. This is also because the viscous drag force is negligibly small in comparison with the Darcy resistance due to the presence of a large particle suspension.

A layer of ferrofluid heated from below in a porous medium has relevance and importance in chemical technology, geophysics and bio-mechanics. In the present analysis the effect of a magnetic field dependent viscosity on the thermal convection in a ferromagnetic fluid in a porous medium is studied.

2. Mathematical Formulation of the Problem

We consider an infinite, horizontal layer of thickness d of an electrically non-conducting incompressible ferromagnetic fluid in a porous medium, heated from below. Its viscosity is $\mu = \mu_1(1+\delta.\mathbf{B})$, μ_1 being the viscosity when the magnetic field is absent and $\mathbf{B}$ the magnetic induction. The magnetic field dependent viscosity appears in ferromagnetic fluids due to the tendency of the magnetic particles to form chains in the presence of an external field. δ has been taken to be isotropic, $\delta_1 = \delta_2 = \delta_3$. Hence $\mu_x = \mu_1(1+\delta B_1)$, $\mu_y = \mu_1(1+\delta B_2)$, $\mu_z = \mu_1(1+\delta B_3)$. A uniform magnetic field H_0 acts along the vertical z-axis. The temperatures at the bottom and top surfaces $z = \mp\frac{1}{2}d$ are T_0 and T_1, and a uniform temperature gradient $\beta = |\mathrm{d}T/\mathrm{d}z|$ is maintained (see Fig. 1). The gravity field $\boldsymbol{g}(0,0,-g)$ pervades the system. The fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity ε and medium permeability k_1, where the porosity is defined as

$$\varepsilon = \frac{\text{volume of the voids}}{\text{total volume}}, (0 < \varepsilon < 1).$$

For very fluffy foam materials, ε is nearly one, and in beds of packed spheres ε is in the range of 0.25 – 0.50.

The equations governing the motion of ferromagnetic fluids in a porous medium for the above model are as follows:

Continuity equation for an incompressible fluid:

$$\nabla.\boldsymbol{q} = 0. \tag{1}$$

Momentum equation for a Darcy model:

$$\begin{aligned}&\frac{\rho_0}{\varepsilon}\left[\frac{\partial}{\partial t}+\frac{1}{\varepsilon}(\boldsymbol{q}\cdot\nabla)\right]\boldsymbol{q}\\ &\quad= -\nabla p+\rho\boldsymbol{g}+\nabla\cdot(\boldsymbol{HB})-\frac{\mu}{k_1}\boldsymbol{q}.\end{aligned} \tag{2}$$

Temperature equation for an incompressible ferromagnetic fluid:

$$\begin{aligned}&\left[\rho_0 C_{\mathrm{V,H}}-\mu_0\boldsymbol{H}.\left(\frac{\partial\boldsymbol{M}}{\partial T}\right)_{\mathrm{V,H}}\right]\frac{\mathrm{d}T}{\mathrm{d}t}+(1-\varepsilon)\rho_\mathrm{s}C_\mathrm{s}\frac{\partial T}{\partial t}\\ &\quad+\mu_0 T\left(\frac{\partial\boldsymbol{M}}{\partial T}\right)_{\mathrm{V,H}}\cdot\frac{\mathrm{d}\boldsymbol{H}}{\mathrm{d}t} = K_1\nabla^2 T+\Phi_T.\end{aligned} \tag{3}$$

Density equation:

$$\rho = \rho_0\left[1-\alpha(T-T_a)\right], \tag{4}$$

where ρ, ρ_0, $\boldsymbol{q}$, t, p, μ, μ_0, $\boldsymbol{H}$, $\boldsymbol{B}$, $C_{\mathrm{V,H}}$, T, $\boldsymbol{M}$, K_1, α, and Φ_T are the fluid density, reference density, filter, velocity, time, pressure, magnetic viscosity, magnetic permeability, magnetic field, magnetic induction, specific heat at constant volume and magnetic field, temperature, magnetization (defined by (6) below), thermal conductivity (assumed constant), thermal expansion coefficient and viscous dissipation function containing second order terms in velocity gradient, respectively. $\boldsymbol{\Phi}_T$ is small and may be neglected. The partial derivatives of $\boldsymbol{M}$ are material properties which can be evaluated, once the magnetic equation of state, such as (8) below, is known. T_a is the average temperature, given by $T_\mathrm{a} = (T_0 + T_1)/2$, where T_0 and T_1 are the constant average temperatures of the lower and upper surfaces of the layer. In writing (2), use has been made of the Boussinesq approximation, and two additional complications are assumed: the viscosity is anisotropic and dependent on the magnetic field.

Maxwell's equations, simplified for a non-conducting fluid with no displacement currents, become

$$\nabla \cdot \boldsymbol{B} = 0, \quad \nabla \times \boldsymbol{H} = \boldsymbol{0}. \tag{5a,b}$$

In the Chu formulation of electrodynamics (Penfield and Haus [18]), the magnetic field $\boldsymbol{H}$, magnetization $\boldsymbol{M}$, and the magnetic induction $\boldsymbol{B}$ are related by

$$\boldsymbol{B} \equiv \mu_0(\boldsymbol{H} + \boldsymbol{M}). \tag{6}$$

We assume that the magnetization is aligned with the magnetic field, but allow a dependence on the magnitude of the magnetic field as well as the temperature:

$$\boldsymbol{M} = \frac{\boldsymbol{H}}{H} M(H, T). \tag{7}$$

The magnetic equation of state, linearized about the magnetic field H_0, and an average temperature T_a, becomes

$$M = M_0 + \chi(H - H_0) - K_2(T - T_\mathrm{a}), \tag{8}$$

where the susceptibility and the pyromagnetic coefficient are defined by

$$\chi = \left(\frac{\partial M}{\partial H}\right)_{H_0, T_\mathrm{a}}, \quad K_2 = -\left(\frac{\partial M}{\partial T}\right)_{H_0, T_\mathrm{a}},$$

where H_0 is the uniform magnetic field of the fluid layer when placed in an external magnetic field $\boldsymbol{H} = \hat{\boldsymbol{k}} H_0^{\mathrm{ext}}$, $\hat{\boldsymbol{k}}$ is the unit vector in the z-direction, H the magnitude of $\boldsymbol{H}$ and $M_0 = M(H_0, T_\mathrm{a})$. Thus the analysis is restricted to physical situations in which the magnetization induced by temperature variations is small compared to that induced by the external magnetic field.

The basic state is assumed to be quiescent and is given by

$$\begin{aligned}
&\boldsymbol{q} = \boldsymbol{q}_\mathrm{b} = (0,0,0), \quad p = p_\mathrm{b}(z), \quad \rho = \rho_\mathrm{b}(z),\\
&T = T_\mathrm{b}(z) = -\beta z + T_a, \quad \beta = \frac{T_1 - T_0}{d},\\
&\boldsymbol{H}_0 = \left[H_0 - \frac{K_2 \beta z}{1+\chi}\right]\hat{\boldsymbol{k}}, \quad \boldsymbol{M}_0 = \left[M_0 + \frac{K_2 \beta z}{1+\chi}\right]\hat{\boldsymbol{k}},\\
&H_0 + M_0 = H_0^{\mathrm{ext}}.
\end{aligned} \tag{9}$$

Only the spatially varying parts of $\boldsymbol{H}_0$ and $\boldsymbol{M}_0$ contribute to the analysis, so that the direction of the external magnetic field is unimportant and the convection is the same whether the external magnetic field is parallel or antiparallel to the gravitational force.

3. The Perturbation Equations and Normal Mode Analysis Method

We shall analyze the stability of the basic state by introducing the following perturbations:

$$\begin{aligned}
&\boldsymbol{q} = \boldsymbol{q}_\mathrm{b} + \boldsymbol{q}', \quad \rho = \rho_\mathrm{b} + \rho', \quad p = p_\mathrm{b}(z) + p',\\
&T = T_\mathrm{b}(z) + \theta, \quad \boldsymbol{H} = \boldsymbol{H}_\mathrm{b}(z) + \boldsymbol{H}',\\
&\boldsymbol{M} = \boldsymbol{M}_\mathrm{b}(z) + \boldsymbol{M}',
\end{aligned} \tag{10}$$

where $\boldsymbol{q}' = (u, v, w)$, p', ρ', θ, $\boldsymbol{H}'$ and $\boldsymbol{M}'$ are perturbations in velocity, pressure, density, temperature, magnetic field and magnetization. These perturbations are assumed to be small. Then the linearized perturbation equations become

$$\frac{\rho_0}{\varepsilon}\frac{\partial u}{\partial t} = -\frac{\partial p'}{\partial x} + \mu_0(M_0 + H_0)\frac{\partial H_1'}{\partial z} - \frac{\mu_1}{k_1}u, \tag{11}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial v}{\partial t} = -\frac{\partial p'}{\partial y} + \mu_0(M_0 + H_0)\frac{\partial H_2'}{\partial z} - \frac{\mu_1}{k_1}v, \tag{12}$$

$$\begin{aligned}
\frac{\rho_0}{\varepsilon}\frac{\partial w}{\partial t} = &-\frac{\partial p'}{\partial z} + \mu_0(M_0 + H_0)\frac{\partial H_3'}{\partial z} - \frac{\mu_1}{k_1}w\\
&- \mu_0 K_2 \beta H_3' + \frac{\mu_0 K_2^2 \beta \theta}{1+\chi} + g\alpha\rho_0\theta\\
&- \frac{\mu_1}{k_1}\delta\mu_0(M_0 + H_0)w,
\end{aligned} \tag{13}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{14}$$

$$\begin{aligned}
&\rho C_1 \frac{\partial \theta}{\partial t} - \mu_0 T_0 K_2 \varepsilon \frac{\partial}{\partial t}\left(\frac{\partial \Phi'}{\partial z}\right)\\
&\quad = K_1 \nabla^2 \theta + \left[\rho C_2 \beta - \frac{\mu_0 T_0 K_2^2 \beta}{(1+\chi)}\right] w,
\end{aligned} \tag{15}$$

where

$$\rho C_1 = \varepsilon\rho_0 C_{\mathrm{V,H}} + (1-\varepsilon)\rho_{\mathrm{S}} C_{\mathrm{S}} + \varepsilon\mu_0 K_2 H_0, \quad \rho C_2 = \rho_0 C_{\mathrm{V,H}} + \mu_0 K_2 H_0. \tag{16}$$

Equations (7) and (8) yield

$$\left.\begin{aligned} H_3' + M_3' &= (1+\chi)H_3' - k_2\theta, \\ H_i' + M_i' &= \left(1+\frac{M_0}{H_0}\right)H_i' \quad (i=1,2). \end{aligned}\right\} \tag{17}$$

Here we have assumed $K_2\beta d \ll (1+\chi)H_0$. Equation (5b) suggests that we can write $\boldsymbol{H}' = \nabla\,\Phi'$, where Φ' is the perturbed magnetic potential.

Eliminating u, v, p' in (11), (12) and (13), using (14), we obtain

$$\begin{aligned} \left(\frac{\rho_0}{\varepsilon}\frac{\partial}{\partial t} + \frac{\mu_1}{k_1}\right)\nabla^2 w = {} & -\mu_0 K_2\beta\left(\nabla_1^2\frac{\partial\Phi'}{\partial z}\right) \\ & +\rho_0 g\alpha(\nabla_1^2\theta) + \frac{\mu_0 K_2^2\beta}{(1+\chi)}(\nabla_1^2\theta) \\ & -\frac{\mu_1}{k_1}\delta\mu_0(M_0+H_0)\nabla_1^2 w, \end{aligned} \tag{18}$$

where $\nabla_1^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ and $\nabla^2 \equiv \nabla_1^2 + \frac{\partial^2}{\partial z^2}$.

From (17), we have

$$(1+\chi)\frac{\partial^2\Phi'}{\partial z^2} + \left(1+\frac{M_0}{H_0}\right)\nabla_1^2\Phi' - K_2\frac{\partial\theta}{\partial z} = 0. \tag{19}$$

We perform a normal mode expansion of the dependent variables in the form

$$(w,\theta,\Phi') = [W(z),\Theta(z),\Phi(z)]\exp[i(k_x x + k_y y)], \tag{20}$$

where k_x, k_y are the wave numbers along the x- and y-directions, respectively, and $k = \sqrt{(k_x^2+k_y^2)}$ is the overall horizontal wave number. $W(z)$, $\Theta(z)$, $\Phi(z)$ are, respectively, the amplitude of z-component of the perturbation velocity, perturbation temperature and perturbation magnetization.

Equations (18), (15), and (19), using (20), become

$$\begin{aligned} & \left(\frac{\rho_0}{\varepsilon}\frac{\partial}{\partial t} + \frac{\mu_1}{k_1}\right)\left(\frac{\partial^2}{\partial z^2} - k^2\right)W \\ & = \frac{\mu_0 K_2\beta}{1+\chi}\left[(1+\chi)\frac{\partial\Phi}{\partial z} - K_2\Theta\right]k^2 \\ & \quad -\rho_0 g\alpha k^2\Theta + \frac{\mu_1}{k_1}k^2\delta\mu_0(M_0+H_0)W, \end{aligned} \tag{21}$$

$$\begin{aligned} & \rho C_1\frac{\partial\Theta}{\partial t} - \mu_0 T_0 K_2\varepsilon\frac{\partial}{\partial t}\left(\frac{\partial\Phi}{\partial z}\right) \\ & = K_1\left(\frac{\partial^2}{\partial z^2} - k^2\right)\Theta \\ & \quad + \left(\rho C_2\beta - \frac{\mu_0 T_0 K_2^2\beta}{1+\chi}\right)W, \end{aligned} \tag{22}$$

$$(1+\chi)\frac{\partial^2\Phi}{\partial z^2} - \left(1+\frac{M_0}{H_0}\right)k^2\Phi - K_2\frac{\partial\Theta}{\partial z} = 0. \tag{23}$$

Equations (21)–(23) give the following dimensionless equations

$$\begin{aligned} & \left(\frac{1}{\varepsilon}\frac{\partial}{\partial t^*} + \frac{1}{k_1^*}\right)(D^2-a^2)W^* \\ & = aR^{1/2}[M_1 D\Phi^* - (1+M_1)T^*] + \frac{a}{k_1^*}\delta^* M_3 W^*, \end{aligned} \tag{24}$$

$$\begin{aligned} & P_r'\frac{\partial T^*}{\partial t^*} - \varepsilon P_r M_2\frac{\partial}{\partial t^*}(D\Phi^*) \\ & = (D^2-a^2)T^* + aR^{1/2}(1-M_2)W^*, \end{aligned} \tag{25}$$

$$D^2\Phi^* - a^2 M_3\Phi^* - DT^* = 0, \tag{26}$$

where the following non-dimensional parameters are introduced:

$$\begin{aligned} & t^* = \frac{\nu}{d^2}t, \quad W^* = \frac{d}{\nu}W, \\ & \Phi^* = \frac{(1+\chi)K_1 aR^{1/2}}{K_2\rho C_2\beta\nu d^2}\Phi, \quad R = \frac{g\alpha\beta d^4\rho C_2}{\nu K_1}, \\ & T^* = \frac{K_1 aR^{1/2}}{\rho C_2\beta\nu d}\Theta, \quad a = kd, \quad z^* = \frac{z}{d}, \\ & D = \frac{\partial}{\partial z^*}, \quad k_1^* = \frac{k_1}{d^2}, \quad P_r = \frac{\nu}{K_1}\rho C_2, \\ & P_r' = \frac{\nu}{K_1}\rho C_1, \quad M_1 = \frac{\mu_0 K_2^2\beta}{(1+\chi)\alpha\rho_0 g}, \\ & \delta^* = \delta\mu_0 H_0(1+\chi), \quad M_2 = \frac{\mu_0 T_0 K_2^2}{(1+\chi)\rho C_2}, \\ & M_3 = \frac{1+M_0/H_0}{1+\chi}. \end{aligned} \tag{27}$$

4. Exact Solution for Free Boundaries

Consider the case where both boundaries are free and perfect conductors of heat. The case of two free boundaries is of little physical interest, but it is mathematically important because they permit to obtain an exact solution, whose properties guide our analysis.

The boundary conditions are

$$W^* = D^2W^* = T^* = D\Phi^* = 0 \text{ at } z = \pm\frac{1}{2}. \quad (28)$$

Following the analysis of Finlayson, the exact solutions satisfying boundary conditions given by

$$\begin{aligned} &W^* = A_1 e^{\sigma t^*} \cos\pi z^*,\ T^* = B_1 e^{\sigma t^*} \cos\pi z^*, \\ &D\Phi^* = C_1 e^{\sigma t^*} \cos\pi z^*,\ \Phi^* = \left(\frac{C_1}{\pi}\right) e^{\sigma t^*} \sin\pi z^*, \end{aligned} \quad (29)$$

where A_1, B_1, C_1 are constants and σ is the growth rate, which is, in general, a complex constant.

Substituting (29) in (24)–(26) and dropping the asterisks for convenience, we get the following three linear, homogeneous algebraic equations in the constants A_1, B_1 and C_1

$$\begin{aligned} &\left[\left(\frac{\sigma}{\varepsilon} + \frac{1}{k_1}\right)(\pi^2 + a^2) + \frac{a^2}{k_1}\delta M_3\right] A_1 \\ &\quad + (aR^{1/2}M_1)C_1 - aR^{1/2}(1+M_1)B_1 = 0, \end{aligned} \quad (30)$$

$$\begin{aligned} &(1-M_2)aR^{1/2}A_1 - (\pi^2 + a^2 + P_r'\sigma)B_1 \\ &\quad + (\varepsilon_r M_2\sigma)C_1 = 0, \end{aligned} \quad (31)$$

$$-\pi^2 B_1 + (\pi^2 + a^2M_3)C_1 = 0. \quad (32)$$

For the existence of non-trivial solutions of the above equations, the determinant of the coefficients of A_1, B_1, C_1 in (30)–(32) must vanish. This determinant on simplification yields

$$V\sigma_i^2 + iW\sigma_i + X = 0, \quad (33)$$

where

$$V = -\frac{(1+x)}{\varepsilon}\left\{(P_r' - \varepsilon P_r M_2) + xP_r'M_3\right\}, \quad (34)$$

$$\begin{aligned} W = &\frac{1}{\varepsilon}(1+x)^2(1+xM_3) \\ &+ \frac{1}{P_\ell}\{1+x+x\delta M_3\}\left\{(P_r' - \varepsilon P_r M_2) + xP_r'M_3\right\}, \end{aligned} \quad (35)$$

$$\begin{aligned} X = &\frac{1}{P_\ell}(1+x)(1+xM_3)(1+x+x\delta M_3) \\ &- R_1 x(1-M_2)\{1+x(1+M_1)M_3\}, \end{aligned} \quad (36)$$

where $R_1 = R/\pi^4$, $x = a^2/\pi^2$, $i\sigma_i = \sigma/\pi^2$, and $P_\ell = \pi^2 k_1$.

5. The Case of Stationary Convection

When the instability sets in as stationary convection (and $M_2 \cong 0$), the marginal state will be characterized by $\sigma_i = 0$. Putting $\sigma_i = 0$, the dispersion relation (33) reduces

$$R_1 = \frac{(1+x)(1+xM_3)\{1+x+x\delta M_3\}}{P_\ell x\{1+x(1+M_1)M_3\}}, \quad (37)$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x, the magnetic parameters M_1 and M_3, the medium permeability parameter P_ℓ and the MFD viscosity δ.

To investigate the effects of the medium permeability, the MFD viscosity and magnetic parameters, we examine the behaviour of $\mathrm{d}R_1/\mathrm{d}P_\ell$, $\mathrm{d}R_1/\mathrm{d}\delta$, $\mathrm{d}R_1/\mathrm{d}M_3$, and $\mathrm{d}R_1/\mathrm{d}M_1$ analytically.

From (37) follows that

$$\begin{aligned} \frac{\mathrm{d}R_1}{\mathrm{d}P_\ell} = &-\frac{1}{P_\ell^2}\Big[(1+x)^2(1+xM_3) + x(1+x)(1+xM_3) \\ &\cdot \delta M_3\Big]\Big[x\{1+x(1+M_1)M_3\}\Big]^{-1}, \end{aligned} \quad (38)$$

$$\frac{\mathrm{d}R_1}{\mathrm{d}\delta} = \frac{1+x}{P_\ell}\frac{(1+xM_3)M_3}{\{1+x(1+M_1)M_3\}}. \quad (39)$$

Thus for a stationary convection, the medium permeability has always destabilizing effect, whereas the MFD viscosity has always stabilizing effect for thermal convection in a ferromagnetic fluid saturating a porous medium.

Equation (37) also yields

$$\begin{aligned} \frac{\mathrm{d}R_1}{\mathrm{d}M_3} = &-\frac{1+x}{P_\ell}\{(1+x)M_1 - \delta[(1+xM_3)^2 + M_1x^2M_3^2]\} \\ &\cdot \{1+x(1+M_1)M_3\}^{-2}, \end{aligned} \quad (40)$$

$$\frac{\mathrm{d}R_1}{\mathrm{d}M_1} = -\frac{(1+x)}{P_\ell}\frac{(1+xM_3)(1+x+x\delta M_3)}{\{1+x(1+M_1)M_3\}^2}. \quad (41)$$

In the absence of MFD viscosity ($\delta = 0$) (which means μ is constant), (40) yields that $\mathrm{d}R_1/\mathrm{d}M_3$ is always negative implying the destabilizing effect of magnetization. In the presence of a MFD viscosity, nothing specific can be said, since the magnetization has a dual role. In the presence of a MFD viscosity, the magnetization M_3 has a destabilizing (or stabilizing effect) if

$$\delta < (\text{or} >) \frac{M_1}{\left((1+xM_3)^2 + M_1x^2M_3^2\right)}, \quad (42)$$

whereas the magnetic parameter M_1 has always a destabilizing effect.

The role of the medium permeability, the MFD viscosity and the magnetic parameters derived and discussed above can also be illustrated with the help of Figures 2 – 6. In Fig. 2, R_1 is plotted against the wave number x for $M_1 = 1000$, $M_3 = 1$, $\delta = 0.05$; $P_\ell = 0.001$, 0.002, 0.003 and 0.004. In Fig. 3, R_1 is plotted against the wave number x for $M_1 = 1000$, $M_3 = 1$, $P_\ell = 0.001$; $\delta = 0.01$, 0.03, 0.05, 0.07. It is clear that the medium permeability hastens the onset of convection, whereas the MFD viscosity postpones the onset of convection as the Rayleigh number decreases and increases with the increase in the medium permeability parameter and the MFD viscosity parameter, respectively. In Fig. 4, R_1 is plotted against the wave number x in the absence of MFD viscosity ($\delta = 0$) for $M_1 = 1000$, $P_\ell = 0.001$; $M_3 = 1$, 3, 5, 7. In Fig. 5, R_1 is plotted against the wave number x in the presence of MFD viscosity for $M_1 = 1000$, $\delta = 0.05$, $P_\ell = 0.001$; $M_3 = 1$, 3, 5, 7. It is clear that the magnetization hastens the onset of convection in the absence of MFD viscosity as the Rayleigh number decreases with the increase in the magnetization parameter, whereas the magnetization hastens (for smaller values of wave numbers) and postpones (for higher values of wave numbers) the onset of convection in the presence of MFD viscosity. It is also observed that, in the absence of MFD viscosity, as the equation of state becomes more non-linear (M_3 large) the fluid layer is destabilized slightly. In Fig. 6, R_1 is plotted against the wave number x for $M_3 = 1$, $\delta = 0.01$, $P_\ell = 0.001$; $M_1 = 0$, 1, 5, 10. It is evident from this figure that the Rayleigh number decreases with increase in the magnetic parameter M_1, thereby showing its destabilizing effect on the system. Thus M_1 hastens the onset of convection, and in its absence, i.e. $M_1 = 0$, higher values of R_1 are needed for the onset of convection.

For sufficiently large values of M_1 we obtain the results for the magnetic mechanism operating in a porous medium

$$N = R_1 M_1 = \frac{(1+x)(1+xM_3)(1+x+x\delta M_3)}{P_\ell x^2 M_3}, \quad (43)$$

where N is the magnetic thermal Rayleigh number.

In Table 1, the critical wave numbers and critical magnetic thermal Rayleigh numbers for the onset of instability are determined numerically using the Newton-Raphson method for the condition $\mathrm{d}N/\mathrm{d}x = 0$. The critical magnetic thermal Rayleigh number (N_c), depends on the magnetization M_3, the medium permeability P_ℓ, and the MFD viscosity δ.

Table 1. Critical magnetic thermal Rayleigh numbers and wave numbers of the unstable modes at marginal stability for the onset of stationary convection.

			$P_\ell = 0.001$	$P_\ell = 0.002$	$P_\ell = 0.003$	$P_\ell = 0.004$
δ	M_3	x_c	N_c	N_c	N_c	N_c
0.01	1	1.99	6794.98	3397.49	2265.00	1698.74
	3	1.44	5181.64	2590.82	1727.21	1295.41
	5	1.28	4827.63	2413.82	1609.21	1206.90
	7	1.20	4685.81	2342.91	1561.94	1171.46
0.03	1	1.98	6884.78	3442.39	2294.93	1721.20
	3	1.41	5361.21	2680.60	1787.07	1340.30
	5	1.24	5089.27	2544.64	1696.42	1272.32
	7	1.14	5026.31	2513.16	1675.44	1256.58
0.05	1	1.97	6974.39	3487.20	2324.80	1743.60
	3	1.38	5539.32	2769.66	1846.44	1384.83
	5	1.19	5347.20	2673.60	1782.40	1336.80
	7	1.09	5360.02	2680.01	1786.67	1340.01
0.07	1	1.96	7063.82	3531.91	2354.61	1765.96
	3	1.36	5716.10	2858.05	1905.37	1429.03
	5	1.16	5601.87	2800.94	1867.29	1400.47
	7	1.05	5688.06	2844.03	1896.02	1422.02
0.09	1	1.94	7153.08	3576.54	2384.36	1788.27
	3	1.33	5891.67	2945.83	1963.89	1472.92
	5	1.13	5853.71	2926.85	1951.24	1463.43
	7	1.01	6011.29	3005.65	2003.76	1502.82

In Fig. 7, N_c is plotted against the MFD viscosity δ for $M_3 = 1$; $P_\ell = 0.001$, 0.002, 0.003 and 0.004. This shows that, as the medium permeability P_ℓ increases, the critical magnetic Rayleigh number (N_c) decreases. Therefore, lower values of N_c are needed for the onset of convection with an increase in P_ℓ, hence justifying the destabilizing effect of the medium permeability P_ℓ. In Fig. 8, N_c is plotted against the MFD viscosity δ for $P_\ell = 0.001$; $M_3 = 3$, 5, 7. This shows that, as the magnetization parameter M_3 increases, the critical magnetic Rayleigh number N_c decreases for lower values of δ and increases for higher values of δ. Therefore lower values of N_c are needed for the onset of convection with an increase in M_3 for lower values of δ, whereas higher values of N_c are needed for the onset of convection with an increase in M_3 for higher values of δ, hence justifying the competition between the destabilizing effect of the magnetization M_3 and the stabilizing effect of the MFD viscosity δ. This can also be observed from the Table 1.

A suggestion of Finlayson [8] has also been taken for the variation of these parametric values. In the present analysis, the range of values pertaining to ferric oxide, kerosene and other organic carriers are chosen. With the same ferric oxide, different carriers like

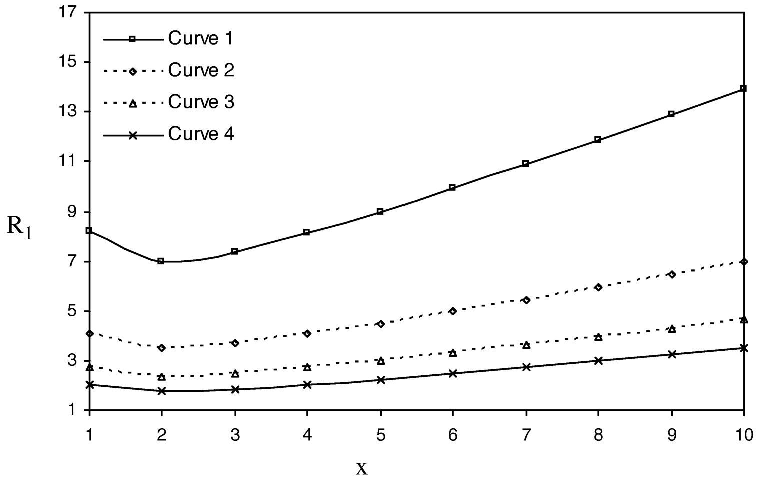

Fig. 2. The variation of Rayleigh number (R_1) with wave number (x) for $M_1 = 1000$, $M_3 = 1$, $\delta = 0.05$; $P_\ell = 0.001$ for curve 1, $P_\ell = 0.002$ for curve 2, $P_\ell = 0.003$ for curve 3 and $P_\ell = 0.004$ for curve 4.

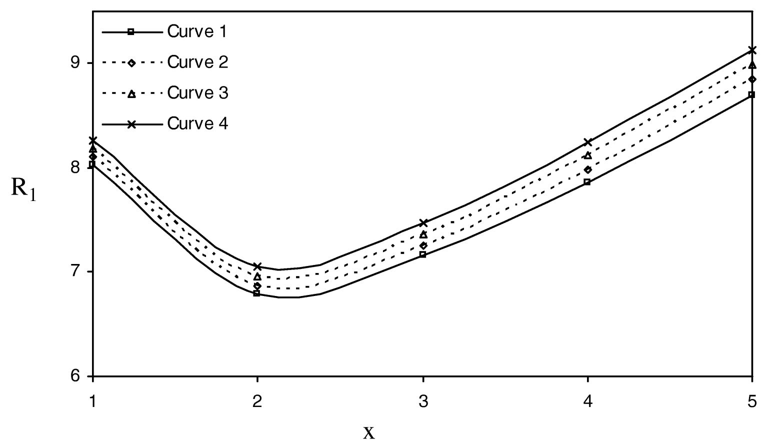

Fig. 3. The variation of Rayleigh number (R_1) with wave number (x) for $M_1 = 1000$, $P_\ell = 0.001$, $M_3 = 1$; $\delta = 0.01$ for curve 1, $\delta = 0.03$ for curve 2, $\delta = 0.05$ for curve 3 and $\delta = 0.07$ for curve 4.

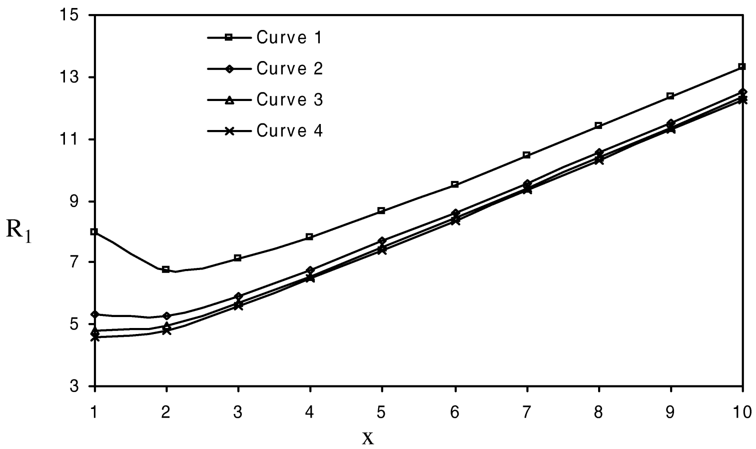

Fig. 4. The variation of Rayleigh number (R_1) with wave number (x) for $M_1 = 1000$, $P_\ell = 0.001$, $\delta = 0$; $M_3 = 1$ for curve 1, $M_3 = 3$ for curve 2, $M_3 = 5$ for curve 3 and $M_3 = 7$ for curve 4.

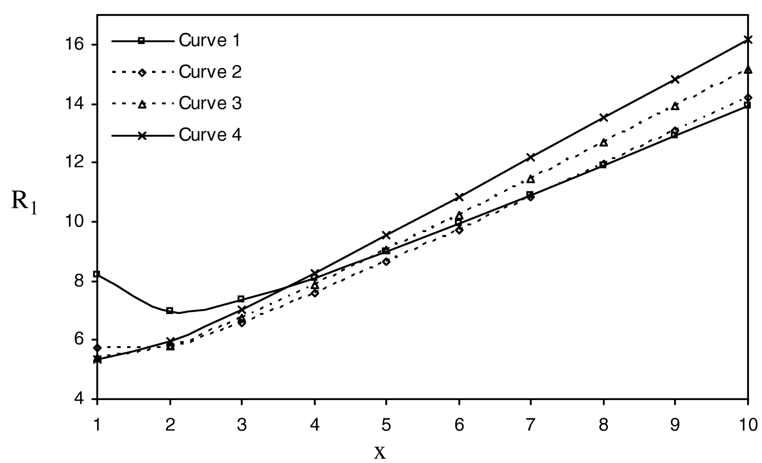

Fig. 5. The variation of Rayleigh number (R_1) with wave number (x) for $M_1 = 1000$, $P_\ell = 0.001$, $\delta = 0.05$; $M_3 = 1$ for curve 1, $M_3 = 3$ for curve 2, $M_3 = 5$ for curve 3 and $M_3 = 7$ for curve 4.

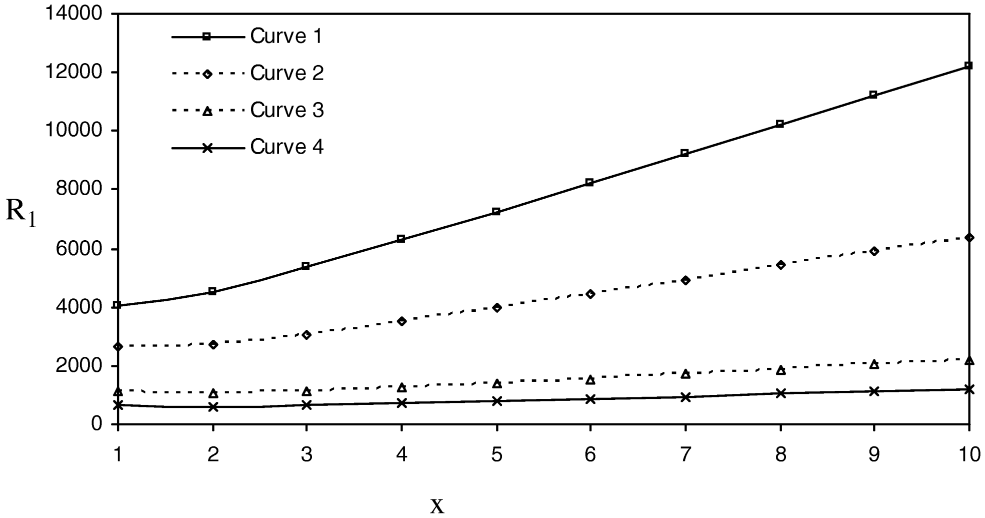

Fig. 6. The variation of Rayleigh number (R_1) with wave number (x) for $M_3 = 1$, $P_\ell = 0.001$, $\delta = 0.01$; $M_1 = 0$ for curve 1, $M_1 = 1$ for curve 2, $M_1 = 5$ for curve 3 and $M_1 = 10$ for curve 4.

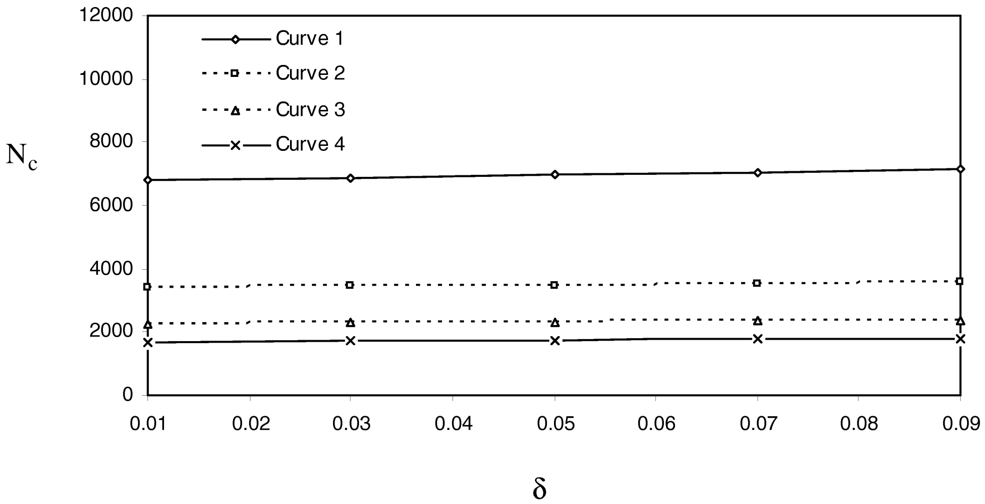

Fig. 7. The variation of critical magnetic Rayleigh number (N_c) with MFD viscosity (δ) for $M_3 = 1$ for $P_\ell = 0.001$ for curve 1, $P_\ell = 0.002$ for curve 2, $P_\ell = 0.003$ for curve 3 and $P_\ell = 0.004$ for curve 4.

alcohol, hydrocarbon, ester, halocarbon, silicon could be chosen. Depending on this, the parametric values of ferromagnetic fluids are found to vary within these limits. For such fluids, M_2 is assumed to have a negligible value and hence is taken to be zero (Sekar and Vaidyanathan [19]). The parameter M_3 measures the departure of linearity in the magnetic equation of state, and values from one ($M_0 = \chi H_0$) to higher values are possible for the usual equation of state, and moreover the higher values of the magnetization parameter M_3 in ferromagnetic fluid has also been taken by several authors (Finlayson [8], Gupta and Gupta [20], Vaidyanathan et al. [21], Sekar and Vaidyanathan [19], Shivakumara et al. [22]). The MFD viscosity δ is increased from 0.01 to 0.09.

6. Principle of Exchange of Stabilities

Here we examine the possibility of oscillatory modes, if any, on the stability problem due to the

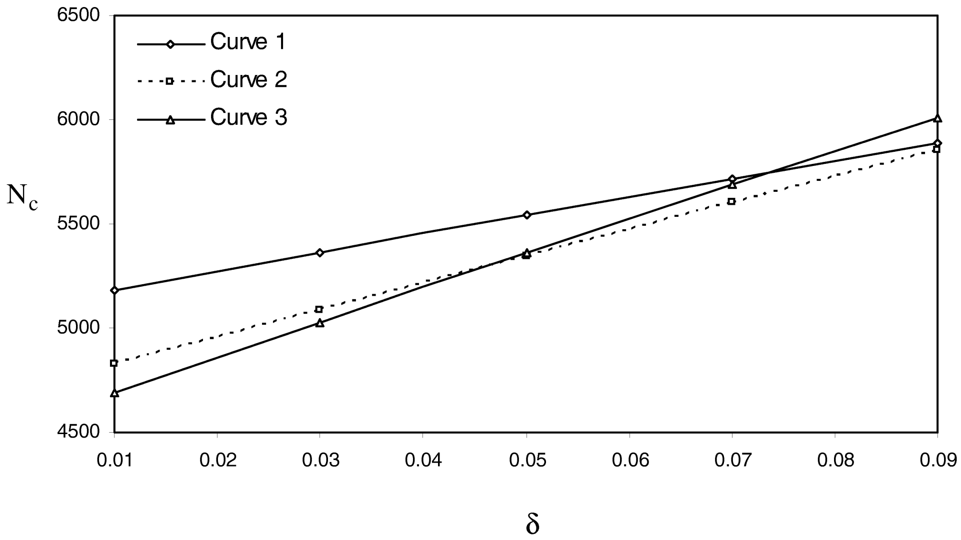

Fig. 8. The variation of critical magnetic Rayleigh number (N_c) with MFD viscosity (δ) for $P_\ell = 0.001$; $M_3 = 3$ for curve 1, $M_3 = 5$ for curve 2 and $M_3 = 7$ for curve 3.

presence of magnetization and medium permeability. Equating the imaginary parts of (33), we obtain

$$\sigma_i \Big[\frac{1}{\varepsilon}(1+x)^2(1+xM_3) + \frac{1}{P_\ell}\{1+x+x\delta M_3\} \cdot \big\{(P_r' - \varepsilon P_r M_2) + xP_r' M_3\big\}\Big] = 0. \tag{44}$$

Here the quantity inside the brackets is positive definite because the typical values of M_2 are $+10^{-6}$ [9]. Hence

$$\sigma_i = 0. \tag{45}$$

This shows that, whenever $\sigma_r = 0$ implies that $\sigma_i = 0$, then the stationary (cellular) pattern of flow prevails on the onset of instability. In other words, the principle of exchange of stabilities is valid for the ferromagnetic fluid heated from below in porous medium.

7. Conclusion and Discussion

In the last millennium, the investigation on the interaction of electromagnetic fields with fluids attracted researchers because of the increase of applications in areas such as chemical reactor, engineering, medicine, high-speed silent printers, etc. Ferrohydrodynamics deals with the interaction of magnetic fields with nonconducting ferromagnetic fluids, which has aroused a lot interest [6]. A layer of a ferrofluid heated from below has relevance and importance in bio-mechanics (e.g. in physiotherapy and in the synthesis of silicone magnetic fluids for use in eye surgery [23]). In armatures of motors and transformers, the coil and core rotates with a finite angular velocity. In this process the viscosity of the ferromagnetic fluid contained in the armatures changes. To compensate this, one has to use ferromagnetic fluids of moderate viscosity ranges in order to have efficient heat transfer. This greatly enables one to choose a proper ferromagnetic fluid for high speed applications. Recently, Jakabský et al. [24] have studied the utilization of ferromagnetic fluids in mineral processing and water treatment. They considered the utilization of ferromagnetic fluids as a separating and modifying medium affecting the magnetic properties of the solid and liquid materials.

In this paper we studied effect of MFD viscosity on the thermal convection in ferromagnetic fluids for fluid layers heated from below saturating a porous medium in the presence of an uniform vertical magnetic field. Using the linearized stability theory and normal mode analysis, an exact solution is obtained for the case of two free boundaries. We have investigated the effects of medium permeability, MFD viscosity and nonlinearity of magnetization (i.e. M_3) on the linear stability. The principal conclusions from the analysis of this paper are:

1. For the case of stationary convection, MFD viscosity has always a stabilizing effect, whereas medium permeability has always a destabilizing effect on the onset of convection. In the absence of MFD viscos-

ity ($\delta = 0$) (which means the viscosity is constant), magnetization has always a destabilizing effect. In the presence of MFD viscosity, nothing specific can be said, since there is competition between the destabilizing role of the magnetization M_3 and the stabilizing role of the MFD viscosity δ. This can also be observed from Fig. 4 (in the absence of MFD viscosity) and Fig. 5 (in the presence of MFD viscosity). In the presence of MFD viscosity, the magnetization M_3 has destabilizing (or stabilizing) effect if $\delta <$ (or $>$)$\frac{M_1}{\left((1+xM_3)^2+M_1x^2M_3^2\right)}$, whereas the magnetic parameter M_1 has always a destabilizing effect. For sufficiently small values of magnetic parameter M_1, the effect of medium permeability, the MFD viscosity and magnetization can also be illustrated with the help of Figs. 2 – 6.

2. The critical wave number and critical magnetic thermal Rayleigh number for the onset of instability are determined numerically for sufficiently large values of the magnetic parameter M_1. Graphs have been plotted by giving numerical values to the parameters, to depict the stability characteristics. It is clear from Table 1 and Fig. 7 that lower values of N_c are needed for the onset of convection with an increase in P_ℓ, hence justifying the destabilizing effect of the medium permeability P_ℓ. It is evident from Table 1 and Fig. 8 that lower values of N_c are needed for the onset of convection with increase in M_3 for smaller values of δ, whereas higher values of N_c are needed for the onset of convection with increase in M_3 for higher values of δ, hence justifying the competition between the destabilizing effect of the magnetization M_3 and the stabilizing effect of the MFD viscosity δ.

3. In the last section we examine the possibility of oscillatory modes. Here we conclude that the principle of exchange of stabilities is valid for ferromagnetic fluids heated from below saturating a porous medium.

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